LXXIII. *Of the best Form of Geographical Maps. By the Rev.* Patrick Murdoch, *M. A. F. R. S.*

Read Feb. 9, 1758.

I. WHEN any portion of the earth's surface is projected on a plane, or transferred to it by whatever method of description, the real dimensions, and very often the figure and position of countries, are much altered and misrepresented. In the common projection of the two hemispheres, the meridians and parallels of latitude do indeed intersect at right angles, as on the globe; but the linear distances are every-where diminished, excepting only at the extremity of the projection: at the center they are but half their just quantity, and thence the superficial dimensions but one-fourth part: and in less general maps this inconvenience will always, in some degree, attend the *stereographic* projection.

The *orthographic*, by parallel lines, would be still less exact, those lines falling altogether oblique on the extreme parts of the hemisphere. It is useful, however, in describing the circum-polar regions: and the rules of both projections, for their elegance, as well as for their uses in astronomy, ought to be retained, and carefully studied. As to Wright's, or Mercator's, nautical chart, it does not here fall under our consideration: it is perfect in its kind; and will always be reckoned among the chief inventions of the last age. If it has been misunderstood, or misapplied, by geographers, they only are to blame.

 II. The

II. The particular methods of defcription propofed or ufed by geographers are fo various, that we might, on that very account, fufpect them to be faulty; but in moft of their works we actually find thefe two blemifhes, *the linear diftances vifibly falfe*, and *the interfections of the circles oblique*: fo that a quadrilateral rectangular fpace fhall often be reprefented by an oblique-angled rhomboid figure, whofe diagonals are very far from equal; and yet, by a ftrange contradiction, you fhall fee a fixed fcale of diftances inferted in fuch a map.

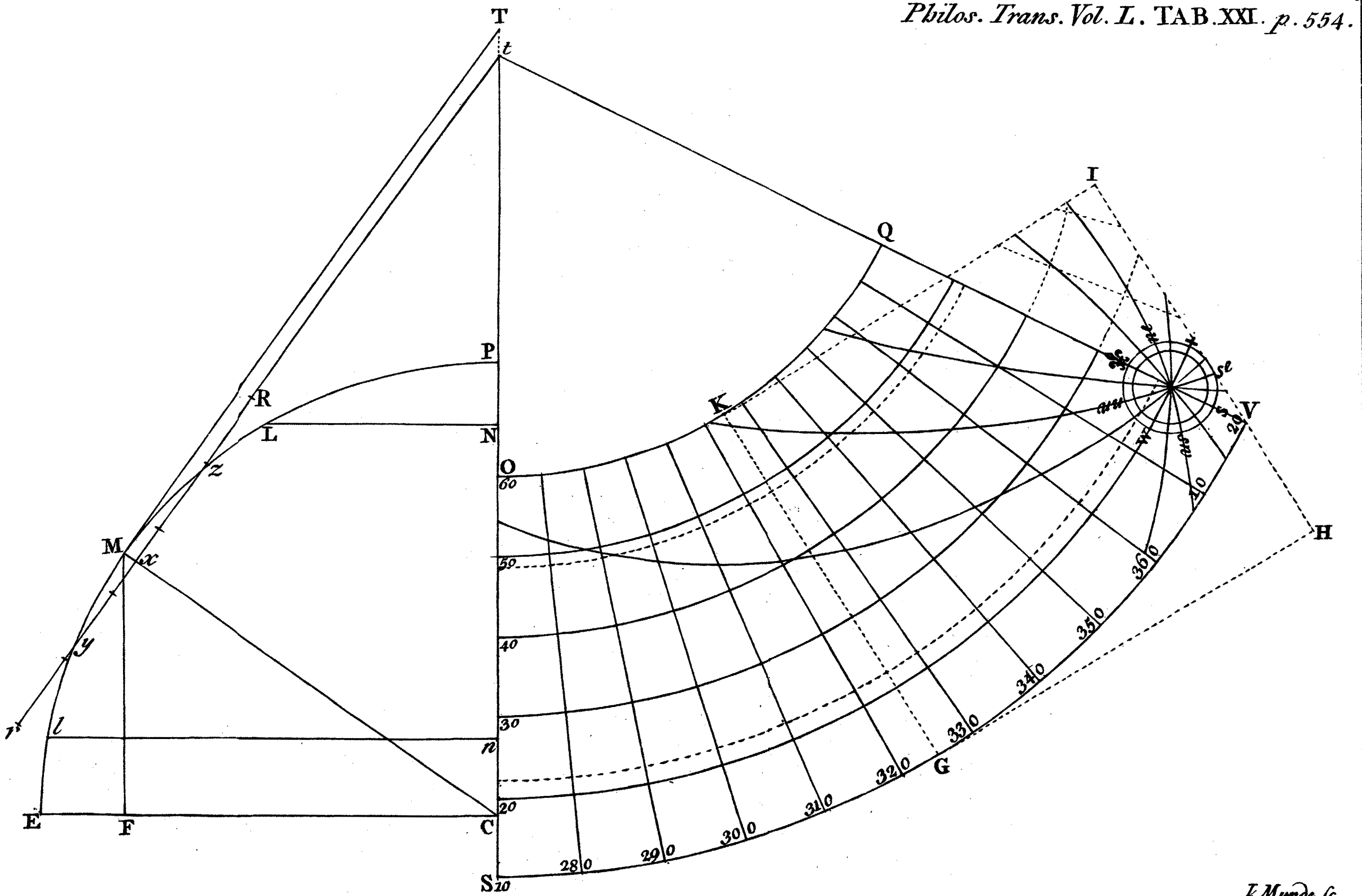
III. The only maps I remember to have feen, in which the laft of thefe blemifhes is removed, and the other leffened, are fome of P. Schenk's of Amfterdam, a map of the Ruffian empire, the Germania Critica of the famous Profeffor Meyer, and a few more ‡. In thefe the meridians are ftraight lines converging to a point; from which, as a center, the parallels of latitude are defcribed: and a rule has been publifhed for the drawing of fuch maps *. But as that rule appears to be only an eafy and convenient approximation, it remains ftill to be inquired, *What is the conftruction of a particular map, that fhall exhibit the fuperficial and linear meafures in their trueft proportions?* In order to which,

IV. Let E*l*LP, in this figure (*See* TAB. XXI.) be the quadrant of a meridian of a given fphere, whofe center is C, and its pole P; EL, E*l*, the latitudes of two places in that meridian, EM their

‡ *Senex* drew feveral of that form.

* See the Preface to the fmall Berlin Atlas.

middle


T
t
Q
I
P
R
L
N
K
V
H
O
z
M
x
y
l
n
E
F
C
S
G
60
50
40
30
20
10
280
290
300
310
320
330
340
350
360
10
20

J. Mynde sc.

middle latitude. Draw LN, ln, cosines of the latitudes, the sine of the middle latitude MF, and its cotangent MT. Then writing unity for the radius, if in CM we take $Cx = \frac{Nn}{Ll \times MF \times MT}$, and thro' x we draw xR, xr, equal each to half the arc Ll, and perpendicular to CM; the conical surface generated by the line Rr, while the figure revolves on the axis of the sphere, will be equal to the surface of the zone that is to be described in the same time by the arc Ll; as will easily appear by comparing that conical surface with the zone, as measured by *Archimedes*.

And, lastly, If from the point t, in which rR produced meets the axis, we take the angle CtV in proportion to the longitude of the proposed map, as MF the sine of the middle latitude is to radius, and draw the parallels and meridians as in the figure, the whole space SOQV will be the proposed part of the conical surface expanded into a plane; in which the places may now be inserted according to their known longitudes and latitudes.

EXAMPLE.

V. Let Ll, the breadth of the zone, be 50°, lying between 10° and 60° north latitude; its longitude 110°, from 20° east of the Canaries to the center of the western hemisphere; comprehending the western parts of Europe and Africa, the more known parts of North America, and the ocean that separates it from the old continent.

And because $Cx = \frac{Nn}{Ll \times MF \times MT}$, add these three logarithms.

 Log.

Log. 0.8726650 (= 50° to radius 1)	— 1.9408476
Log. MF (fin. 35°)	— 1.7585913
Log. MT (tang. 55°)	0.1547732
Take the fum	— 1.8542121
from log. Nn (= .6923772)	— 1.8403427
the remainder	— 1.9861306

is the logarithm of Cx. And becaufe 1 : Cx :: MT : xt, to this adding the log. MT 0.1547732

The fum 0.1409038

is the log. of xt = 1.383260; and xR (= xr = $\frac{1}{2}$ Ll) being .4363325, Rt will be 0.9469275, rt = 1.8195925. Whence having fixed upon any convenient fize for our map, the center t is eafily found. As, allowing an inch to a degree of a great circle, or 50 inches to the line Rr, Rt the femidiameter of the leaft parallel will be 54.255 inches, and that of the greateft parallel 104.255 inches.

Again, making as radius to MF fo the longitude 110° to the angle StV, that angle will be 63° 5′$\frac{3}{5}$. Divide the meridians and parallels, and finifh the map as ufual.

Note, The log. MT being repeated in this computation with a contrary fign, we may find xt immediately by fubtracting the fum of the logarithms of Ll and MF from the log. of Nn.

VI. A map drawn by this rule will have the following properties:

1. The interfections of the meridians and parallels will be rectangular.

2. The

2. The diſtances north and ſouth will be exact; and any meridian will ſerve as a ſcale.

3. The parallels thro' z and y, where the line Rr cuts the arc Ll, or any ſmall diſtances of places that lie in thoſe parallels, will be of their juſt quantity. At the extreme latitudes they will exceed, and in mean latitudes, from x towards z or y, they will fall ſhort of it. But unleſs the zone is very broad, neither the exceſs nor the defect will be any-where conſiderable.

4. The latitudes and the ſuperficies of the map being exact, by the conſtruction, it follows, that the exceſſes and defects of diſtance, now mentioned, compenſate each other; and are, in general, of the leaſt quantity they can have in the map deſigned.

5. If a thread is extended on a plane, and fixed to it at its two extremities, and afterwards the plane is formed into a pyramidal or conical ſurface, it may be eaſily ſhewn, that the thread will paſs thro' the ſame points of the ſurface as before; and that, *converſely*, the ſhorteſt diſtance between two points in a conical ſurface is the right line which joins them, when that ſurface is expanded into a plane. Now, in the preſent caſe, the ſhorteſt diſtances on the conical ſurface will be, if not equal, always nearly equal, to the correſpondent diſtances on the ſphere: and therefore, all rectilinear diſtances on the map, applied to the meridian as a ſcale, will, nearly at leaſt, ſhew the true diſtances of the places repreſented.

6. In maps, whoſe breadth exceeds not 10° or 15°, the rectilinear diſtances may be taken for ſufficiently exact. But we have choſen our example of a greater breadth than can often be required, on pur-

poſe

pose to shew how high the errors can ever arise; and how they may, if it is thought needful, be nearly estimated and corrected.

Write down, in a vacant space at the bottom of the map, a table of the errors of equidistant parallels, as from five degrees to five degrees of the whole latitude; and having taken the mean errors, and diminished them in the ratio of radius to the sine of the mean inclination of the line of distance to the meridian, you shall find the correction required; remembering only to distinguish the distance into its parts that lie *within* and *without* the sphere, and taking the difference of the correspondent errors, in *defect* and in *excess*.

But it was thought needless to add any examples; as, from what has been said, the intelligent reader will readily see the use of such a table; and chiefly as, whenever exactness is required, it will be more proper, and indeed more expeditious, to compute the distances of places by the following canon.

Multiply the product of the cosines of the two given latitudes by the square of the sine of half the difference of longitude; and to this product add the square of the sine of half the difference of the latitudes; the square root of the sum shall be the sine of half the arc of a great circle between the two places given.

Thus, if we are to find the true distance from one angle of our map to the opposite, that is, from S to Q, the operation will be as follows:

L. sin.

$$
\begin{array}{rl}
\text{L. fin. } 30^\circ = & -\ 1.6989700 \\
\text{L. fin. } 80^\circ = & -\ 1.9933515 \\
2\ \text{L. fin. } 55^\circ = & -\ 1.8267290 \\
\hline
 & -\ 1.5190505 = \text{log. of } 0.330408 \\
\text{and } 2\ \text{L. fin. } 25^\circ = & -\ 1.2518966 = \text{log. of } 0.178606
\end{array}
$$

Log. of the fum . . . 0.509014 is — 1.7067297
Whole half is — 1.8533648
the L. fin. of 45° 31′, the double of which is 91° 2′, or 5462 geographical miles.

And feeing the lines TS, TQ, reduced to minutes of a degree, are 6255.189 and 3255.189 refpectively. and the angle STV is $63^\circ\ 5'\frac{3}{7}$, the right line SQ on the map will be 5594′, exceeding its juft value by 132′ or $\frac{1}{42}$ of the whole.

7. The errors on the parallels increafing faft towards the north, and the line SQ having, at laft, nearly the fame direction, it is not to be wondered that the errors in our example fhould amount to $\frac{1}{42}$. Greater ftill would happen, if we meafured the diftance from O to Q by a ftraight line joining thofe points: for that line, on the conic furface, lying every-where at a greater diftance from the fphere than the points O and Q, muft plainly be a very improper meafure of the diftance of their correfpondent points on the fphere. And therefore, to prevent all errors of that kind, and confine the other errors in this part of our map to narrower bounds, it will be beft to terminate it towards the pole by a ftraight line KI touching the parallel OQ in the middle point K, and on the eaft and weft by lines, as HI, parallel to the meridian thro' K, and meeting the tangent at the middle point of the parallel SV in H. By this means too we fhall gain more fpace than we lofe, while the map takes the ufual

rectangular

rectangular form, and the spaces GHV remain for the *title*, and other inscriptions.

VII. Another, and not the least considerable, property of our map is, that it may, without sensible error, be used as a sea-chart; the rumb-lines on it being logarithmic spirals to their common pole t, as is partly represented in the figure: and the arithmetical solutions thence derived will be found as accurate as is necessary in the art of sailing.

Thus if it were required to find the course a ship is to steer between two ports, whose longitudes and latitudes are known, we may use the following

RULE.

To the logarithm of the number of minutes in the difference of longitude add the constant logarithm * — 4.1015105, *and to their sum the logarithm sine of the mean latitude, and let this last sum be* S.

The cotangent of the mean latitude being T, *and an arithmetical mean between half the difference of latitude and its tangent being called* m, *from the logarithm of* T + m *take the logarithm of* T — m, *and let the logarithm of their difference be* D; *then shall* S — D *be nearly the logarithm tangent of the angle, in which the ship's course cuts the meridians.*

Note, We ought, in strictness, to use the ratio of $tx + xR$ to $tx - xR$ instead of $T + m$ to $T - m$; but we substitute this last as more easily computed, and very little different.

* This constant logarithm contains the reduction of the diff. of longitude to parts of radius unity, and to *Briggs*'s Modulus.

EXAMPLE

EXAMPLE I.

Let the latitudes, on the ſame ſide of the equator, be 10° and 60°; then the middle latitude and its complement are 35° and 55°, and half the difference of the latitudes is 25°: and the difference of longitude being 110°, the operation will ſtand as below.

Log. 6600′ (in 110°) . . 3.8195439
Conſtant log. . . . —4.1015105

—1.9210544
Log. ſin. 35° . . . —1.7585913

S = ... —1.6796457

Again T = 1.4281480
m = .4513202

Log. $\overline{T+m}$ (= 1.8794682) 0.2740350
Log. $\overline{T-m}$ (= 0.9768278) —1.9898180

Log. 0.2842170 = D = —1.4536500

S — D (= log. tangent 59° 16′) . . . . . = 0.2259957
agreeing to a minute with the ſolution by a table of meridional parts.

EXAMPLE 2.

The reſt remaining, let the difference of longitude be only 40°; then

Log. 2400′ (in 40°) . . 3.3802112
Conſtant log. . . . —4.1015105

—1.4817217
Log. ſin. 35° . . . —1.7585913

S = ... —1.2403130
D (as before) = —1.4536500

S — D (= log. tang. 31° 27′ ½) . . . —1.7866630

EXAMPLE 3.

Let the difference of longitude be 40°; but the latitudes 56° and 80°;

And log. 2400′ } + log. constant } = − 1.4817217
Log. ſin. 68° . . . = − 1.9671659

S = ... − 1.4488876

T (tang. 22°) = .4040262
m = ...2109980

Log. $\overline{T + m}$ (= .6150242) − 1.7888921
Log. $\overline{T - m}$ (= .1830282) − 1.2625181

Log. . . 0.5263740 = D = − 1.7212944

S − D (= log. tangent 28° 6′) . . . . = − 1.7275932
wanting of the true anſwer no more than 1° 4′.

And in all caſes that can occur, the error of this rule will be inconſiderable.

It is not meant, however, that it ought to take place of the eaſier and better computation by a table of meridional parts: but it was thought proper to ſhew, by ſome examples, how ſafely the map itſelf may be depended on in the longeſt voyages; provided it is ſufficiently large, and the neceſſary rumb-lines are exactly drawn *.

* See *Coteſii* Logometr. prop. 6.

"last age. If it has been misunderstood or misap-
"plied by geographers, they only are to blame."—
And again, at the end of his nautical examples, he concludes thus, *viz.* "It is not meant, however, "that it ought to take place of the easier and "better computation by a table of meridional "parts."

I have the honour to be, with the greatest respect,

SIR,

The ROYAL SOCIETY's, and

Your most obedient Servant,

William Mountaine.

ADDENDA *to Mr.* Murdoch's *Paper*, N°. LXXIII.

IF it is required "to draw a map, in which the superficies of a "given zone shall be equal to the zone on the sphere, while "at the same time the projection from the center is strictly geo-"metrical;" *Take* Cx *to* CM *as a geometrical mean between* CM *and* Nn, *is to the like mean between the cosine of the middle latitude, and twice the tangent of the semidifference of latitudes*; and project on the conic surface generated by *xt*. But here the degrees of latitude towards the middle will fall short of their just quantity, and at the extremities exceed it: which hurts the eye. Artists may use either rule: or, in most cases, they need only make C*x* to CM as the arc ML is to its tangent, and finish the map; either by a projection, or, as in the first method, by dividing that part of *xt* which is intercepted by the secants thro' L and *l*, into equal degrees of latitude.

Mr. Mountaine justly observes, "that my rule does not admit of "a zone containing N. and S. latitudes." But the remedy is, *to extend the lesser latitudes to an equality with the greater; that the cone may be changed into a cylinder, and the rumbs into straight lines.*